A Probabilistic Approach to Monitoring Maintenance in Aviation Projects

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Abstract—The assessment of equipment reliability during operation and deployment gives a clear idea of the replacement of its component as stipulated, at different time intervals. Thus the prediction of maintenance needs, to ensure desirable level of equipment reliability is of great importance to maintenance personnel. The case studied in this project work illustrates the application and use of an operational availability model that is based on aircraft level information extracted from in-service maintenance data of a fleet aircraft, currently in operation. The data is categorized into four major failure categories, viz. repairable failure (avionics), replaceable failure with delay due to sourcing of spares(avionics), repairable failure (mechanical systems), and replaceable failure with delay due to sourcing of spares(mechanical systems). Detection of a failure from the types given above, affects the aircraft availability and an inspection is carried out by the maintenance personnel to decide the type of maintenance. After the inspection, the repairable failures (avionics, mechanical systems) are dealt immediately. In the case of replaceable failures (avionics, mechanical systems), a delay due to sourcing of spares, is observed due to various reasons. It is this phase of maintenance that seriously affects the aircraft availability.

Thus, to understand the present level of maintenance with the objective to devise a better maintenance system to reduce the aircraft downtime, due to the reasons stated above, the optimum reliability indices such as MTSF (mean time to system failure), steady state aircraft availability are estimated numerically, using semi-Markov process and regenerative point techniques. Graphs are plotted to demonstrate the results.

Index Terms—Repair, replacement, failure, fleet aircraft, equipment, semi-Markov and regenerative point technique.

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1 INTRODUCTION

A complex system like an aircraft is made up of sophisticated equipment, which has increased the demand to identify failure prone parts and define planned maintenance programs. Basically, the concept of reliability studies the reliability of critical parts to quantitatively predict the performance of the equipment. Thus, this project discusses a steady state operational availability model which can be used to meet aircraft fleet management requirements and is based on in-service maintenance data including delay times for sourcing spares and allows for impact analysis. The predictive capability of this model is able to provide the aircraft fleet with a moreaccurate maintenance analysis decision support capability.

Engineering systems under different operational situations and circumstances have been analyzed by a number of researchers [2], [3], [4], [5], [6], [7] &[8]. Recently, Mundhir*et.al* [1] wrote about attaining zero failure performance for GIV gulfstream aircraft through reliability modeling and analysis at a fleet aircraft operations centre. Considerable research has been done in this domain due to its ability to identify flaws in system design, compare several possible system configurations, minimize downtime, maximize operational readiness, reduce operating costs and develop optimum maintenance

policies.

With a few exceptions, it is noted that most of the authors have discussedthe theoretical aspects than the real practical side of the work i.e. including inspection and delay times due to sourcing of spares. Furthermore, when real data was used there is little discussion on its origin and the broader implications of the results. Thus, there is a considerable gap between theory and applications.

Noting the above, this project attempts to bridge this gap by studying a fleet aircraft presently operative with a fleet operator in Muscat, Oman. The model is developed from real in-service maintenance data of the aircraft. Mathematical techniques like semi–Markov and regenerative processes are used to obtain the reliability indices which are used in the evaluation process and validation of the model. The case specific details of the aircraft are used in the modeling study and depiction of graphical results.

The aircraft's operational availability is hindered due to any one of the four types of failure as seen from the data, i.e. repairable failure (avionics), replaceable failure with delay due to sourcing of spares(avionics), repairable failure (mechanical systems), and replaceable failure with delay due to sourcing of spares (mechanical systems). The failed unit/componentis attended and inspected by the aviation maintenancedepartmentas soon as malfunction/failure is detected. Delays are noted during the sourcing of spares for replacements (avionics, mechanical systems) only. The unit/component regenerates and works like new after each repair or replacement.

The collected data gives the following estimations:
Probability of repairable failure (avionics) p_1= 0.61111.
Probability of replaceable failure (avionics) p_2= 0.38888.
Probability of repairable failure (mechanical systems) p_3=

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0.507936507.

Probability of replaceable failure (mechanical systems) p_4= 0.492063492.

Estimated value of failure rate λ = 0.0001807403123 per hour.

Estimated value of repair rate (avionics) α_1= 1.205479452 per hour.

Estimated value of replacement rate with delay due to sourcing of spares (avionics) α_2= 0.2812 per hour.

Estimated value of repair rate (mechanical systems) β_3= 0.761904761 per hour.

Estimated value of replacement rate with delay due to sourcing of spares (mechanical systems) β_4 = 0.3265 per hour.

The fleet aircraft is analyzed using semi Markov process and regenerative point technique, and the following reliability indices pertaining to aircraft efficiency are obtained:

• Mean time to aircraft failure.
• Steady state aircraft fleet availability.

2 MODEL DETAILS AND ASSUMPTIONS

The aircraft unit/component is initially operative at state 0 and transits probabilistically subject to the type of failure to any of the five states 1 to 5 with failure rate λ, probabilities p_1, p_2, p_3 and p_4 respectively (refer Figure. 1). An inspection is done at state 1 prior to maintenance decision making.

1. All failure times are assumed to have exponential distribution with failure rate (λ) whereas the repair times have general distributions.
2. After each repair, replacement at state's 2 to 5, the aircraft unit/component works like new and returns back to state 0.
3. Breakdowns are self-announcing.

The aviation maintenance department attends, as soon as a failure is reported/detected.

3 NOTATIONS USED

O   Operative aircraft unit/component.

λ constant failure rate of the aircraft unit/component.

p_1 probability of aircraft unit/component failure (repairable failure avionics).

p_2 probability of aircraft unit/component failure (replaceable failure avionics).

p_3 probability of aircraft unit/component failure (repairable failure mechanical systems).

p_4 probability of aircraft unit/component failure (replaceable failure mechanical systems).

$r_{AV}(t)$ aircraft unit/component is under repair (avionics).

$rep_{AV}(t)$ aircraft unit/component is under replacement (avionics).

$r_{ME}(t)$ aircraft unit/component is under repair (mechanical systems).

$rep_{ME}(t)$ aircraft unit/component is under replacement (mechanical systems).

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$p_{ij}, Q_{ij}(t)$	p.d.f., c.d.f. of first passage time from a regenerative state i to j or to a failed state j in (0, t].
$\varphi_i(t)$	c.d.f. of first passage time from a regenerative state i to a failed state j.
*	Laplace Transforms (LT), i.e., for any f(t) and (t); $f(t) * g(t) = \int_0^t f(t-u)g(u)du$
$g_2(t), G_2(t)$	p.d.f., c.d.f. of repair time of failed aircraft unit/component (avionics).
$g_3(t), G_3(t)$	p.d.f., c.d.f. of replacement time (with delay due to sourcing of spares) of failed aircraft unit/component (avionics).
$g_4(t), G_4(t)$	p.d.f., c.d.f. of repair time of failed aircraft unit/component (mechanical systems).
$g_5(t), G_5(t)$	p.d.f., c.d.f. of replacement time (with delay due to sourcing of spares) of failed aircraft unit/component (mechanical systems).

4 TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A transition diagram showing the different states of transition of the aircraft is as shown in fig. 1. The epochs of entry into states 0, 1, 2, 3, 4 and 5 are regeneration points and hence the states are regenerative states. The states 1, 2, 3, 4 and 5 are failed states. The transition probabilities are as given below:

$dQ_{01}(t) = \lambda e^{-\lambda t}dt$
$dQ_{12}(t) = p_1\alpha e^{-\alpha t}$
$dQ_{13}(t) = p_2\alpha e^{-\alpha t}$
$dQ_{14}(t) = p_3\alpha e^{-\alpha t}$
$dQ_{15}(t) = p_4\alpha e^{-\alpha t}$

$$dQ_{20}(t) = \alpha_1 e^{-\alpha_1 t}$$
$$dQ_{30}(t) = \alpha_2 e^{-\alpha_2 t}$$
$$dQ_{40}(t) = \beta_3 e^{-\beta_3 t}$$

$dQ_{50}(t) = \beta_4 e^{-\beta_4 t}$ (1)- (9)

The non-zero elements p_{ij} are as given below:

p_{01}=1
p_{12}= p_1
p_{13}= p_2
p_{14}= p_3
p_{15}= p_4
p_{20}=1
p_{30}=1
p_{40}=1
p_{50}=1   (10)-(18)

By these transition probabilities it is verified that:

p_{01}=1
$p_{12}+ p_{13}+ p_{14}+ p_{15}= p_1+ p_2+ p_3+ p_4$=1
$p_{20}= p_{30}= p_{40}= p_{50}$=1   (19)-(21)

The mean sojourn time (μ_i) in the regeneration state 'i' is called as the time of stay in that state before transition to any

other state. If 'T' shows the sojourn time in the regenerative state i, then:

$$\mu_i = E(T) = \Pr[T > t]dt$$

Thus: $\mu_0 = \int\limits_0^\infty e^{-\lambda t} dt = \dfrac{1}{\lambda};$

$$\mu_1 = \int_0^\infty \overline{H}(t)dt$$

$$\mu_2 = \int\limits_0^\infty \overline{G}_2(t)dt;$$

$$\mu_3 = \int\limits_0^\infty \overline{G}_3(t)dt;$$

$$\mu_4 = \int\limits_0^\infty \overline{G}_4(t)dt;$$

$$\mu_5 = \int_0^\infty \overline{G}_5(t)dt \qquad (22)\text{-}(27)$$
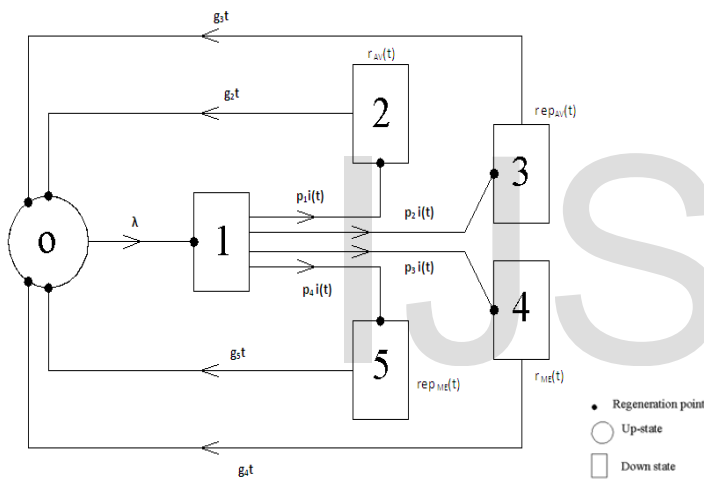

Fig. 1. Transition states of the aircraft

The unconditional mean time taken by the system to change into regenerative state 'j' when it is counted from the epoch of entrance into state 'i' is mathematically stated as:

$$m_{ij} = \int_0^\infty tdQ_{ij}(t) = -q_{ij}*'(0)$$

Thus, $m_{01} = \mu_0$

$m_{12} + m_{13} + m_{14} + m_{15} = \mu_1$

$m_{20} = \mu_2$

$m_{30} = \mu_3$

$m_{40} = \mu_4$

$m_{50} = \mu_5 \qquad (28)\text{-}(33)$

5 MATHEMATICAL ANALYSIS

Mean time to aircraft failure

Regarding the failed states as absorbing states and employing the arguments used for regenerative processes, the following recursive relation for $\phi_i(t)$ is obtained:

$$\emptyset_0(t) = Q_{01}(t) \qquad (34)$$

Solving the above equation for $\phi_0{}^{**}(s)$ by taking Laplace Stieltje's Transforms and using the determinant method, the following is obtained:

$$\phi_0{}^{**}(s) = \frac{N(s)}{D(s)} \qquad (35)$$

Where, N(s)=$Q_{01}{}^{**}$(s) and D(s)=1

Now the mean time to system failure (MTSF) when the unit started at the beginning of state 0, is:

$$\text{MTSF} = \lim_{s \to 0} \frac{1 - \phi_0{}^{**}(s)}{s} = \frac{N}{D} \qquad (36)$$

Where, N=μ_0 and D=1

Aircraft availability analysis

Using the probabilistic arguments and by defining $A_i(t)$ as the probability that the aircraft is in upstate at the instant t, given that the aircraft entered the regenerative state i at t = 0, the following recursive relations are obtained:

$A_0(t) = M_0(t) + q_{01}(t) \copyright A_1(t)$

$\quad A_1(t) = q_{12}(t) \copyright A_2(t) + q_{13}(t) \copyright A_3(t) + q_{14}(t) \copyright A_4(t)$
$\qquad\qquad + q_{15}(t) \copyright A_5(t)$

$A_2(t) = q_{20}(t) \copyright A_0(t)$

$A_3(t) = q_{30}(t) \copyright A_0(t)$

$A_4(t) = q_{40}(t) \copyright A_0(t)$

$A_5(t) = q_{50}(t) \copyright A_0(t) \qquad (37)\text{-}(42)$

Where $M_0(t) = e^{-\lambda t}$.

Taking the Laplace Transforms (L.T.) of the equations shown above and solving them for $A_0 * (s)$, it is got:

$$A_0 * (s) = \frac{N_1(s)}{D_1(s)} \qquad (43)$$

The steady state availability of the aircraft fleet is given as:

$$A_0 = \lim_{s \to 0} s A_0 * (s) = \frac{N_1}{D_1} \qquad (44)$$

Where:

$N_1 = \mu_0$

$D_1 = \mu_0 + \mu_1 + p_1\mu_2 + p_2\mu_3 + p_3\mu_4 + p_4\mu_5$

Table 1. Summary of the data

SL NO	TYPE	FAILURE	NO OF TIMES	TIME TO REPAIR IN HRS	COST IN GBP/HR
01	repairable failure avionics	minor repairs - captive	44	36.5	1800
02	replaceable failure avionics	major replacements - captive	28	68	80896
03	repairable failure mechanical systems	minor repairs - captive	32	42	2150
04	replaceable failure mechanical systems	major replacements - captive	31	55.5	60094

6 PARTICULAR CASE

The following particular case is considered for graphical analysis:

$g_2(t) = \alpha_1 e^{-\alpha_1 t}$

$g_3(t) = \alpha_2 e^{-\alpha_2 t}$

$g_4(t) = \beta_3 e^{-\beta_3 t}$

$g_5(t) = \beta_4 e^{-\beta_4 t}$

$p_{01} = 1$, $p_{12} = p_1$, $p_{13} = p_2$, $p_{14} = p_3$, $p_{15} = p_4$

$p_{20} = 1$, $p_{30} = 1$, $p_{40} = 1$, $p_{50} = 1$

$\mu_0 = \frac{1}{\lambda}$, $\mu_1 = \frac{1}{\alpha}$, $\mu_2 = \frac{1}{\alpha_1}$, $\mu_3 = \frac{1}{\alpha_2}$, $\mu_4 = \frac{1}{\beta_3}$, $\mu_5 = \frac{1}{\beta_4}$

Using the numerical values calculated from the collected data as shown in table 1 and the expressions (35) and (43); the mean time to aircraft failure and aircraft fleet availability is estimated as:

Mean Time to Aircraft Failure: 5532.800001 hours

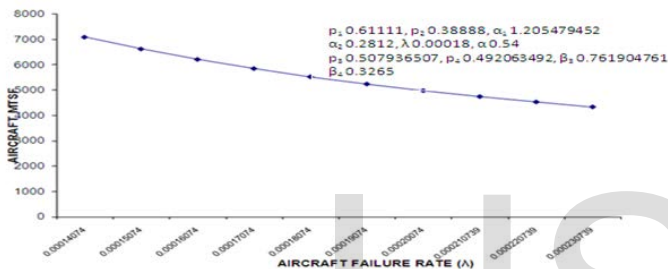**Aircraft Fleet Availability**: 0.998931977

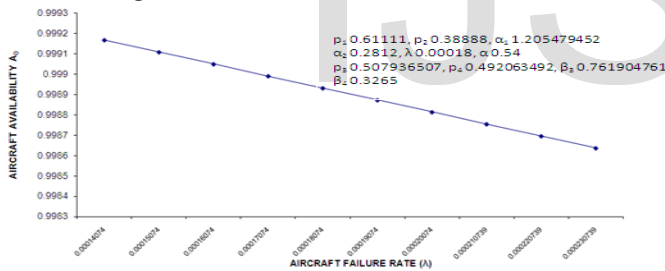**Fig. 2. Aircraft MTSF vs failure rate (λ)**

Fig. 3. Aircraft availability (A₀) vs failure rate (λ)

7 CONCLUSION

Optimum reliability indices of the aircraft are obtained in order to understand the overall system effectiveness and gives useful inputs about the system behaviour. Compared to Mundhir *et. al* [1], this project work has carried out the analysis, wherein the maintenance practices include: inspection (α) and delay time (g_3, g_5) due to the sourcing of spares. In Mundhir *et. al* [1], the values of MTSF and availability (A_0) are: 5555.555556 hours and 0.999460474, whereas in this case it is: 5532.800001 hours, 0.998931977, which clearly shows the impact and effect of inspection and delay time due to the sourcing of spares.

Graphs of MTSF and availability with respect to the failure rate shows a normal declining trend as failure rate increases figure 2, figure3).

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